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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Fn | | (fg)(x) = f(g(x)), composite | | | | | | | | | | | | | | | | Domain: {x| x and f(x) } | | | | | | | | | | | | | | | |
| domain: , range: {x|x } = = [0,) | | | | | | | | | | | | | | | | diff: A \ B = {x|x and x } | | | | | | | | | | | | | | | |
| fn is increasing if a < b f(a) < f(b) for any a, b | | | | | | | | | | | | | | | | fn is decreasing if a < b f(a) > f(b) for any a, b | | | | | | | | | | | | | | | |
| even fn: f(-x) = f(x)  odd fn: f(-x) = -f(x) (if not odd/even, proof by counter e.g.)  power fn: , n | | | | | | | | | | | | | | | | symmetric about y-axis  symmetric about origin  if n is odd, fn is odd | if n is even, fn is even | | | | | | | | | | | | | | | |
| Trigo | | sin2 + cos2 = 1  1 + tan2 = sec2  1 + cot2 = csc2 | | | | | | | | sin(A ± B) = sinA cosB ± cosA sinB  cos(A ± B) = cosA cosB sinA sinB  tan(A ± B) = | | | | | | | sin (-x) = - sin x  cos(-x) = cos(x)  tan(-x) = - tan x | | | | | | | | | | | | | | | csc(-x) = - csc(x)  sec(-x) = sec(x)  cot(-x) = - cot(x) | |
| 1 - x2 ≤ cos x for -π/2 < x < π/2  x < tan x for 0 < x < π/2 | | | | | | | | | If a,b,c are sides of triangle and is angle opp c, then c2 = a2 + b2 -2abcos | | | | | | | | | | | | | | | | | | | -|| ≤ sin ≤ ||  -|| ≤ 1 - cos ≤ || | | | |
| sin 2 = 2sincos =  cos 2 = cos2 - sin2 =  tan 2 = | | | | | | | | | | csc 2 =  sec 2 =  cot 2 = | | | | | | | | | cos2 =  sin2 = | | | | | | | | | | = 1  = 0  = 1 | | |
| sin(π - x) = sin x  cos(π - x) = -cos x  tan(π - x) = -tan x | | | | | | | | | (a+b)3 = a3 + 3a2b + 3ab2 + b3  (a-b)3 = a3 - 3a2b + 3ab2 - b3  y-x = (y1/n-x1/n)(y(n-1)/n + y(n-2)/nx1/n +...+ x(n-1)/n) | | | | | | | | | | | | | | | | | | | =  = | | | |
| Limits | | | | slope = gradient = m = | | | | | | | | | | | | | | tangent line at : ( | | | | | | | | | | | | | | | |
| Intuitive defn | | | | only depends on values of f(x) for x near a, (not at a) | | | | | | | | | | | | | | , if value of f(x) is arbitrarily close to L by taking x sufficiently close to a (intuitive definition) | | | | | | | | | | | | | | | |
| Properties | | | | If and  1.  2.  3. | | | | | | | | | | | 4.  5.  6. , n  7. , n and if n is even, f(x) for all x near a | | | | | | | | | | | | | | | | | | |
| Finding limit | | | | | If a is in domain of f, then | | | | | | | | | | | | | If a not in domain, try simplifying/rationalise fraction | | | | | | | | | | | | | | | |
| 1-sided lim | | | |  | | | | | | | | | | | | | | if DNE | | | | | | | | | | | | | | | |
| Infinite lim | | | | If taking x sufficiently close to a, value of f(x) is arbitrarily large/small,  Infinite limits **make sense**, but **do not exist** | | | | | | | | | | | | | | | | | | | |  | | | | | | | | | |
| Squeeze Theorem | | | | | | | | | If f(x) ≤ g(x) ≤ h(x) for all x near a (except at a) and = L, then exists and = L | | | | | | | | | | | | | | | | | | | | | | | | |
|  | | | | = , where g(x)/h(x) = f(x) | | | | | | | | | | | | | | Use if a not in domain of f(x) | | | | | | | | | | | | | | | |
| Precise defn | | | | , if for every > 0, a > 0 such that |f(x) - L| < whenever 0 < |x-a| < | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Triangle Inequality | | | | | | | | For any a,b , |a|-|b| |a+b| |a|+|b| | | | | | | | | | | | | | | | | | | | | | | | | | |
| Precise defn | | | | Right hand limit: for every > 0, there exists a num > 0 s.t. 0 < x-a < => |f(x) - L| <  Left hand limit: for every > 0, there exists a num > 0 s.t. 0 < a-x < => |f(x) - L| <  + limit: for every > 0, there exists a num > 0 s.t. 0 < |x-a| < => f(x) > M  limit: for every < 0, there exists a num > 0 s.t. 0 < |x-a| < => f(x) < M | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Precise defn as x | | | | x : for every > 0, there exists a num M s.t. x > M => |f(x) - L| <  x : for every > 0, there exists a num M s.t. x < M => |f(x) - L| < | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Prove limits | | | | 1. Let > 0  2. Choose a (found in working) to prove that 0 < |x-a| < => |f(x)-L| < | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Continuous at a pt | | | | Let f be a polynomial or rational fn  If f has the direct substitution property at a, i.e. , then f is continuous at a  Opp of continuous is discontinuous | | | | | | | | | | | | Defn of continuity:  1. f(a) is well-defined, i.e. a is in domain of f; and  2. exists, i.e. it is a real number; and  3. , precise defn of limits apply here as well | | | | | | | | | | | | | | | | | |
| Removable discontinuity | | | | | 2. true but 1. false only at pt a, then f have removable discontinuity  Let f1(x) = . Then f1 is the *continuous extension* of f at a | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Infinite discontinuity | | | | | Suppose f has at least 1 1-sided infinite limit at a: or  Then vertical line x = a is an asymptote of y = f(x) and f is said to have an *infinite discontinuity* at a | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Jump discontinuity | | | | | Suppose and exists, but ≠  Then f is said to have a *jump discontinuity* at a | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1-sided continuity | | | | = f(a). Then f is continuous from the left at a  = f(a). Then f is continuous from the right at a  f is continuous at a iff f is continuous from the left at a and from the right at a | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Continuity on intervals | | | | f is continuous on [a,b]  if f is continuous at every x (a,b), and f is continuous from the right at a, and f is continuous from the left at b | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Proving | | | | Use limits to show that no matter value of a, pt 3 holds | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Composite fn | | | | Let f and g be 2 fn. Suppose = b and = c. Let y = f(x) and z = g(y).  We know (x ≠ a) y and y (y ≠ b) z. But = c is true only if  1. 1st part change to (x ≠ a) y (y ≠ b) OR 2. 2nd part change to z | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Condition 1 means = b and f(x) ≠ b x in an open interval containing a except at a  = c = | | | | | | | | | | | | | | Condition 2 means g is continuous at b  = c = g(b) = g | | | | | | | | | | | | | | | |
| Substitution in limits | | | | - = , where h = x - a. (Derived from condition 1 of composite fn, x (x ≠ a) h (h ≠ 0))  - In particular, if f is continuous at a = f(a) = f(a) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Continuous composite fn | | | | | Suppose f is continuous at a and g is continuous at f(a). Then gf is continuous at a  = = g(f(a)) | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Root fn | | | | Root fn, is continuous on (-∞,∞) if n is odd | | | | | | | | | | | | | | [0,∞) if n is even | | | | | | | | | | | | | | | |
| Trigo fn | | | | sin x and cos x are continuous on  tan x = (sin x / cos x) and sec x = (1 / cos x) are continuous whenever cos x ≠ 0 \ {±π/2, ±3π/2, ±5π/2,...} | | | | | | | | | | | | | | | | cot x = cos x / sin x and  csc x = 1 / sin x are continuous whenever sin x ≠ 0  \ {0, ±π, ±2π, ±3π,...} | | | | | | | | | | | | | |
| IVT | | | | Let f be a continuous fn on [a,b]  Suppose f(a) < 0 and f(b) > 0 or f(a)> 0 and f(b)<0, then c (a,b) s.t. f(c) = 0  Suppose f(a) ≠ f(b) and N is btw f(a) and f(b), then c (a,b) s.t. f(c) = N  Only prove there can be more than 1 root | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Derivative | | | | Slope/gradient = m = = = f'(a) = = = f(a) =  Since = =  Suppose f'(a) exists, then tangent line at x = a: y = f'(a)(x-a) + f(a)  If f is differentiable at a, then f'(a) = , then f is continuous at a  Converse may not be true. f is continuous at a does not imply f is differentiable at a | | | | | | | | | | | | | | | | | | | | | | | | | | | | | f differentiable at a means exists |
| Formulas | | | | |  |  |  |  | | --- | --- | --- | --- | | (cf)' = cf' | (xn) = nxn-1, if n < 0, x cannot be 0 | (sin x) = cos x | (cot x) = -csc2x | | (f ± g)' = f' ± g' | (g-1)' = (1/g)' = -g'/g2 | (cos x) = -sin x | (sec x) = sec x tan x | | (f2)' = 2f f' | = , assuming g(x) ≠ 0 | (tan x) = sec2x | (csc x) = -csc x cot x | | (fg)' = f'g + fg' |  |  | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Chain rule | | | | = | | | | | | If f differentiable at x, g differentiable at f(x), then gf is differentiable at x and (gf)'(x) = g'(f(x))f'(x) | | | | | | | | | | | | | | | | | | | | | | | |
| Implicit differentiation | | | | | | Implicit dy/dx assume dy/dx exists (cannot prove differentiability) | | | | | | | | | | | | E.g. x3 + y3 = 3xy 3x2 + 3y2  = 3y + 3x | | | | | | | | | | | | | | | |
| EVT | | | | Suppose f is continuous on [a,b], c, d [a,b] s.t. f(c) ≤ f(x) ≤ f(d) x [a,b] | | | | | | | | | | | | | | | | | | | | | | | | Extreme values: abs max, min | | | | | |
| Fermat's Theorem | | | | If f has local extreme value at c, then either f'(c) don't exist or f'(c) exist and = 0  This c is called a critical point. (Stationary pt if f'(c) = 0) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Closed Interval Mtd | | | | Let f be continuous on [a,b]  1. Evalute values of f at endpoints: f(a) and f(b)  2. Evaluate values of f at critical points on (a,b) s.t. f'(c) don't exist or s.t. f'(c) = 0  3. Compare values obtained in 1 and 2 (largest value = absolute max; smallest = absolute min) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Rolle's Theorem | | | Suppose fn f is continuous on [a,b] and differentiable on (a,b) and f(a) = f(b)  There must be c (a,b) s.t. f'(c) = 0 | | | | | | | | | | | | | | | | | | | | | | | | If f'(x) = 0 x (a,b), then f(x) = C x (a,b), where C is constant  If f'(x) = g'(x) x (a,b), then f(x) = g(x) + C x (a,b), where C is constant | | | | | | |
| MVT | | | | Suppose fn f is continuous on [a,b] and differentiable on (a,b)  c (a,b) s.t. f'(c) = | | | | | | | | | | | | | | | | | | | | | | |
| Increasing Test | | | | Let fn f be continuous on close interval I, differentiable on interior of I and f'(x) > 0 ( < 0 ) for every x in interior of I  Then, f is increasing (decreasing) on I  If f is non-decreasing on I, then f'(x) ≥ 0 x I | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1st derivative test | | | | Let fn f be continuous at critical pt c, differentiable on open interval containing c except at c  If f' changes from -ve to +ve at c, local min at c. If f' change from +ve to -ve at c, local max  If f' don't change sign at c, no local extreme val at c | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2nd derivative test | | | | Lemma. If exists and +ve (-ve), then f(x) > 0 ( < 0 ) x in an open interval containing a, except at a  Suppose f'(c) = 0. If f''(c) > 0, f has local min at c. If f''(c) < 0, f has local max at c  2nd derivative test is inconclusive if f'(c) = f''(c) = 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Concavity | | | | Suppose f is differentiable on open interval I, where f(x) - f(c) = f'(c)(x-c)  If graph of f lies above all its tangent lines on I, f concave up on I, f(b) - f(a) > f'(a)(b-a) and f' increasing on I  If graph of f lies below all its tangent lines on I, f concave down on I, f(b) - f(a) < f'(a)(b-a) and f' decreasing on I | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Concavity Test | | | | Suppose f is twice differentiable on open interval I  If f''(x) > 0 x I f concave up on I. If f''(x) < 0 x I f concave down on I  If f''(x) = 0, inconclusive | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Inflection point | | | | If f continuous at c, and change concavity at c inflection pt at c  If f is twice differentiable at c, f''(c) = 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| l'Hôpital's Rule | | | | Assume fn f,g are differentiable at a and = f(a) = 0 and = g(a) = 0. Then f,g are continuous at a  = = = = , provided g'(a) ≠ 0 (simple version) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Let fn f,g s.t. = 0 and = 0 and exists or = ±. (Does not matter if a is finite or infinite)  Then = (0/0 version) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Let fn f,g s.t. = and = and exists or ±  Then = (/ form) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Cauchy's MVT | | | | Suppose f,g are continuous on [a,b] and differentiable on (a,b) and g'(x) ≠ 0 for any x (a,b)  Then c (a,b) s.t. =  (Generalized MVT) Let g(x) = x. Then g'(x) = 1 x . Then c (a,b) s.t. f'(c) = | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|  | | Suppose f' exist and is cts on [a,b] and f'' exists on (a,b) where a < b. Then c (a,b) s.t. f(b) = f(a) + (b-a)f'(a) +f''(c) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Area under graph | | | | Let f be non-negative continuous fn on [a,b]  1. Divide [a,b] into n equal subintervals  2. Construct rectangle on each subinterval whose height is value of f at left/right endpoint of subinterval | | | | | | | | | | | | | | | 3. Find total area of n rectangles, Ln / Rn­  4. Ln / Rn approaches actual area as n | | | | | | | | | | | | | | |
| Definite Integral | | | | | | | = | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Integrability of Continuous Fn | | | | | | | If fn f is continuous over interval [a,b], or if f has a finite num of jump discontinuities, then exists and f is integrable over [a,b] | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Geometric properties | | | | Let f be nonnegative continuous fn on [a,b]  Then = area btw y = f(x) and x-axis from a to b  Let f be a nonpositive continuous fn on [a,b]  Then -f is nonnegative and continuous on [a,b]  = area btw y = f(x) and x-axis from a to b | | | | | | | | | | | | | | | | Let f be continuous fn on [a,b]. Let A1 represent area above x-axis and A2 area below x-axis  = A1 - A2 = net area of region btw y = f(x) and x-axis from a to b  = A1 + A2 = area of region | | | | | | | | | | | | | |
|  | 1. = c(b-a)  2. If f(x) ≥ g(x) x [a,b], then ≥  3. Let m,M be min,max val of f on [a,b], then m(b-a) ≤ (x) dx ≤ M(b-a)  4. + = | | | | | | | | | | | | | | | | | | | | | | | | 5. = -  6. If f is defined at a, = 0  7. = c  8. (x) + g(x) dx = (x) dx + (x) dx | | | | | | | | |
| Anti-derivative | | | | F is antiderivative of f on interval I if F'(x) = f(x) x I   |  |  |  | | --- | --- | --- | | 1. xn = xn+1 + C, n ≠ 1 | 5. csc2 kx = - cot kx + C | 9. ∫ cot kx = ln|sin kx| + C | | 2. sin kx = - cos kx + C | 6. sec kx tan kx = sec kx + C | 10. ∫ sec kx = ln|sec kx + tan kx| + C | | 3. cos kx = sin kx + C | 7. csc kx cot kx = - csc kx + C | 11. ∫ csc kx = - ln|csc kx + cot kx| + C | | 4. sec2 kx = tan kx + C | 8. ∫ tan kx = - ln|cos kx| + C |  | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Fundamental Theorem of Calculus | | | | | Suppose f is continuous on [a,b]. Let g(x) = dt  Then g is continuous on [a,b], differentiable on (a,b) and g'(x) = f(x) x (a,b)  = F(b) - F(a) = F(x) | | | | | | | | | | | | | | | | | | | | | | | | = f(x)  = f(u) | | | | |
| MVT for definite integrals | | | | | | | | | | If f is continuous on [a,b], then c in [a,b] s.t. f(c) = | | | | | | | | | | | | | | | | | | | | | | | |
| Indefinite Integrals | | | | | | | | Collection of all antiderivatives of f is called the indefinite integral of f w.r.t x and is denoted by dx | | | | | | | | | | | | | | | | | | | | | | | | | |
| Substi-tuition Rule | | | | Suppose u = g(x) is differentiable, whose range is interval I. Suppose g' is continuous and f is continuous on I. Then(g(x))g'(x) dx = du  And (g(x))g'(x) dx = du | | | | | | | | | | | | | | | | | | E.g. dx. Let u = 1-x2. = -2x  dx = dx = - du = - + C = - (1-x2)3/2 + C | | | | | | | | | | | |
| Odd/ Even Fn | | | | | Let f be continuous fn on [-a,a] | | | | | | | | If f is odd, then dx = 0 | | | | | | | | | | | | | If f is even, then dx = 2dx | | | | | | | |
| Disconti-nuous Fn | | | Let f be continuous on [a,b), discontinuous at b from left, dx = dx  Let f be continuous on (a,b], discontinuous at a from right, dx = dx  dx is convergent is limit exist / divergent is limit DNE | | | | | | | | | | | Suppose f is discontinuous at c (a,b), and  dx = dx + dx  dx is convergent if both integrals exists,  divergent if at least one integral is divergent  Let f be fn continuous on (a,b) anddx and dx exist  Let f1 be the continuous extension of f, then dx = dx | | | | | | | | | | | | | | | | | | | |
| Infinity | | | | If dx exists t ≥ a, then dx = dx  If dx exists t ≤ b, then dx = dx  Integral is convergent if limits exist else divergent | | | | | | | | | | | | | | | | | | | dx =dx dx  Convergent if both improper integrals on right are convergent, else divergent | | | | | | | | | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1-1 fn | Fn is 1-1 (one-to-one) if horizontal line intersects graph ≤ 1 time  OR a ≠ b f(a) ≠ f(b) for any a,b in domain, D of f  OR f(a) = f(b) a = b for any a, b D  Let f be 1-1 fn. Then f-1(y) = x y = f(x). | | | | | | | | f : D R, f-1 : R D. R is domain of f-1 and D is range of f-1  (f -1)-1 = f  f-1(f(x)) = x for any x D. f(f-1(y)) = y for any y R  In general, f-1  f ≠ f f-1 | | | |
| Finding inverse | | | | | f and f-1are symmetric (reflection) w.r.t line y = x | | | | 1. Express x in terms of y, x = f-1(y)  2. Interchange x and y to express f-1 as fn in x, y = f-1(x) | | | |
| Properties | | | | | f is cts. f is 1-1 f is monotonic (incring or decring)  f is 1-1. f is continuous f-1 is continuous | | | | f is 1-1 cts fn. f incring (decring) f-1 incring (decring)  (f-1)'(b) = , f'(a) ≠ 0 | | | |
| Inverse Trigo | | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | fn | domain | range | continuous | differentiable | derivative | | sin-1 | [-1, 1] | [-π/2, π/2] | [-1,1] | (-1,1) | sin-1 x = , x (-1,1) | | cos-1 | [-1, 1] | [0, π] | [-1,1] | (-1,1) | cos-1 x = , x (-1,1) | | tan-1 |  | (-π/2, π/2) |  | | tan-1 x = | | cot-1 |  | (0, π) |  | | cot-1 x = | | sec-1 | (-∞, -1] [1, ∞) | [0, ) [π, ) | (-∞, -1] [1, ∞) | (-∞, -1) (1, ∞) | sec-1 x = , |x| > 1 | | csc-1 | (-∞, -1] [1, ∞) | (0, ] (π, ] | (-∞, -1] [1, ∞) | (-∞, -1) (1, ∞) | csc-1 x = , |x| > 1 | | | | | | | | | | | |
| |  |  |  | | --- | --- | --- | | sin-1 x + cos-1 x = π/2, x [-1,1] | tan-1 x + cot-1 x = π/2 | sec-1 x + csc-1 x = | | | | | | | | | | | |
| Logarithmic fn | | | | | |  |  |  | | --- | --- | --- | | ln x = (x > 0) | ln x: (0, ∞) | ln x is cts, differentiable, incring and concave down on | | ln 1 = 0 | = –∞, = ∞, | Let x > 0 and a > 0. Then ln(ax) = ln a + ln x | | Let x > 0 and r . Then ln(xr) = r ln x | | For any x ≠ 0, ln|x| = and dx = ln|x| + C | | | | | | | | |
| Logarithmic differentiation | | | | | | 1. Take abs value: |y| = |f1(x)|r1|fn(x)|rn  2. Take natural log: ln|y| = r1 ln|f1(x)|+ + rn ln|fn(x)| | | | | | 3. Differentiate w.r.t x  Cannot use logarithmic differentiation if y = 0 | |
| Exponential fn | | | | | Euler's num = e = 2.71828 and  Let n . Then en = e\*e...\*e and e-n = 1/en | | | | ex = exp(x)= f-1: where f(x) = ln x  Let x . Then f(ex) = ln(ex) = x ln e = x = eln x | | | |
| |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | |  | ln e = 1 | e0= 1 | | er = em/n = | = 0 | = ∞ | |  | exey = ex+y | | | (ex)y = exy | e-x = 1/ex | ex = ex | | Let a > 0 | ar = er ln a | | axay = ax+y | (ax)y = axy | a-x = 1/ax | ax = axln a | | xa = axa-1 (x > 0, and for any a) | | | | dx =  If a , then domain of f(x) = xa is [0, ∞] | | e = | | | | | | | | |
| To find f(x)g(x), where f(x) > 0 | | | | | 1. Express f(x)g(x) = exp[g(x)ln f(x)]  2. Interchange lim and exp function | | |
| Hyperbolic Trigo fn | | | | Hyperbolic sine fn: sinh x =  Hyperbolic cosine fn: cosh x =  cosh2 x - sinh2x = 1 | | | | | sinh(x + y) = sinh x cosh y + cosh x sinh y  cosh(x + y) = cosh x cosh y + sinh x sinh y  sinh x = cosh x and cosh x = sinh x | | | |
| |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | fn | shape | domain | range | continuous | differentiable | derivative | | sinh-1 | increasing |  | |  | | sinh-1 x = | | cosh-1 | increasing, concave up | [1,∞) | [0,∞) | [1,∞) | (1,∞) | cosh-1 x = for x > 1 | | | | | | | | | |
| Inverse substitution rule | | | | | | | Let f be continuous fn. Suppose x = g(t) is 1-1 and g' is continuous, then f(x) dx = f(g(t))g'(t) dt | | Just substitution but expanding the integral | | | |
| Integration by parts | | | | | | | | ∫ u dx = uv - ∫ v dx | ∫ u dv = uv - ∫ v du | | | |
|  | | | | | | | | = | n∫ (cos x)n dx = (cos x)n-1sin x + (n-1)∫ (cos x)n-2 dx | | | |
| Trigo sub | | | 1. (a > 0). Let x = a sin t, t [-π/2, π/2]  2. (a > 0). Let x = a tan t, t (-π/2, π/2)  3. (a > 0). Let x = a sec t, t [0, π/2) [π, 3π/2) | | | | | | = a cos t  = a sec t  = a tan t | | | |
| Integration by partial fractions | | | | | | | Just factorise denominator into product of real linear factors and real irreducible quadratic factors | | | | | + + , x6 - 1 = (x-1)(x+1)(x2 + x + 1)(x2 - x + 1) |
| Universal trigo sub | | | | | Let f be a rational expression in 2 var.  ∫ f(sin x, cos x) dx, -π < x < π, can be evaluated by  t = tan(x/2), i.e. x = 2tan-1 t. = | | | | sin x = , cos x =  ∫ f(sin x, cos x) dx = ∫ f(, ) dt | | | |

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| Applications | |  |  |  | | --- | --- | --- | | Let f be a cts fn on [a, b], f(a) = c, f(b) = d, f ≥ g, f-1 ≥ g-1 (if fns on diff side of axis, just take lower one as 0/left as 0) | | | | y = f(x), x = f-1(y) | Rotate abt x-axis | Rotate abt y-axis | | Area | (x) - g(x) dx | (y) - g-1(y) dy | | Volume | (x) dx | (y) dy | | Solids of Revolution (Disk, perp to axis of revolution) | [f(x)2 - g(x)2] dx | [f-1(y)2 - g-1(y)2] dy | | Solids of Revolution (Shell, parallel to axis of revolution) | y[f-1(y) - g-1(y)] dy | x[f(x) - g(x)] dx | | Arc Length | dx | dy | | Surface Area | dx | dy | |

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| ODE | |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | Name | Standard form | | Eqn | | | | | 1st order ODE | = f(x) | | y = ∫ f(x) dx | | | | | = g(y) | | = x = ∫ dy | | | | | 1st order separable ODE | = f(x)g(y) | | ∫ f(x) dx = ∫ dy, (g(y) ≠ 0) | | | | | 1st order homogeneous ODE | = F(x, y)  where F(tx, ty) = F(x, y), i.e. have y/x in F(x, y) | | 1. Let z = . Then y = xz and = z + x.  F(x, y) = F(1, z) for x ≠ 0  2. ODE becomes z + x = F(1, z), which is separable | | | | | 1st order linear ODE | + p(x)y = q(x) | | 1. Evaluate ∫ p(x) dx = P(x) + C  2. Use integrating factor eP(x). Then eP(x) = p(x)eP(x)  3. Multiply integrating factor to eqn [eP(x)y] = eP(x)q(x)  4. y = ∫ eP(x)q(x) dx | | | | | Bernoulli's Eqn | + p(x)y = q(x)yn | | For n ≠ 0, 1. Let z = y1-n. = (1-n)y-n  Multiply (1-n)y-n to DE, (1-n)y-n + (1-n)y-np(x)y = (1-n)y-nq(x)yn  + (1-n)p(x)z = (1-n)q(x), (linear ODE) | | | | | Exponential growth & decay | = ky | | y = Cekt | | | If k > 0: law of natural growth  If k < 0: law of natural decay | | Continuously compounded interest | Annually: A(t) = A0(1+r)t  n times per year: A(t) = A0(1+)nt | | Continuously compounded: Let n ∞, A(t) = = A0ert, r = interest per annum, A0 = initial amt, t = num of years | | | | | Radiocarbon Dating | = km | | m(t) = m(0)ekt, k =  half-life: t1/2 = time for half of qty to decay | | | | | Logistic Population Growth  M = carrying capacity | = kP (no M)  = k(M-P)P,  M > 0, k > 0 (Bernoulli's DE) | P(t) = P0ekt (if k is constant)  P(t) = (logistic fn)  P(0) =  P = at t = | | | k < 0, P(t) 0; k > 0 P(t) ∞ | | | Newton's Law of Cooling | = -r(T-TS) (heat transfer model) where r > 0 | | | T(t) = TS + (T0 - TS)e-rt. As t ∞, T(t) TS | | | |

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